Time reversal invariant gapped boundaries of twisted Z_2 gauge theory

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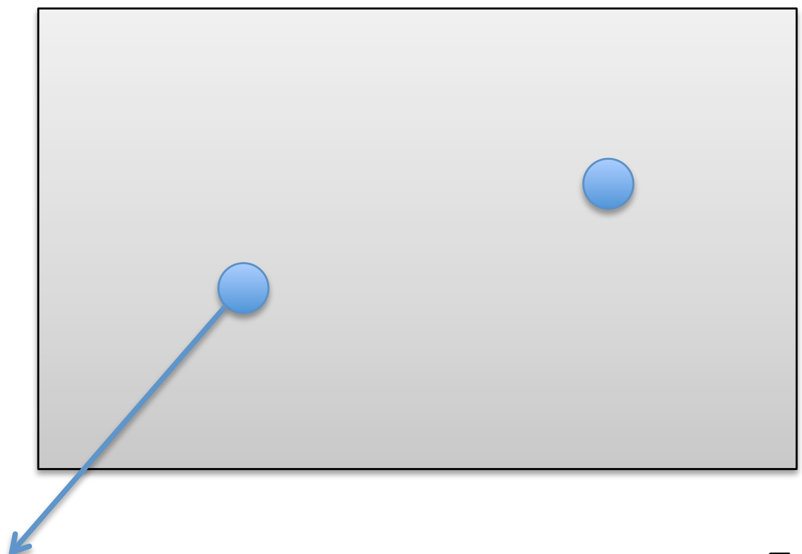
Joint work with Fiona Burnell, Alexei Kitaev, Max Metlitski, Ashvin Vishwanath

Topology + Symmetry + Interaction

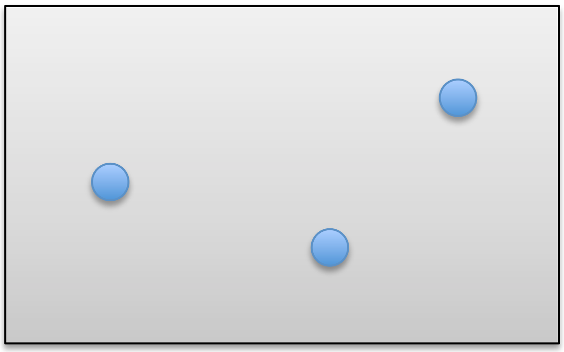


Fractional excitation + Symmetry → Fractional symmetry representation

- Symmetry fractionalization

Symmetry fractionalization

- Fractional quantum Hall



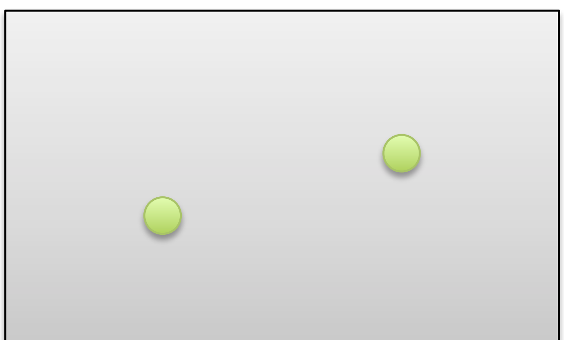
quasi-particle $\frac{1}{3}\text{electron}$

+ charge conservation

$\rightarrow \frac{1}{3}$ electron charge

Laughlin, Stormer, Tsui, 1982

- Z_2 Spin liquid



spinon $\frac{1}{2}\text{spin}$

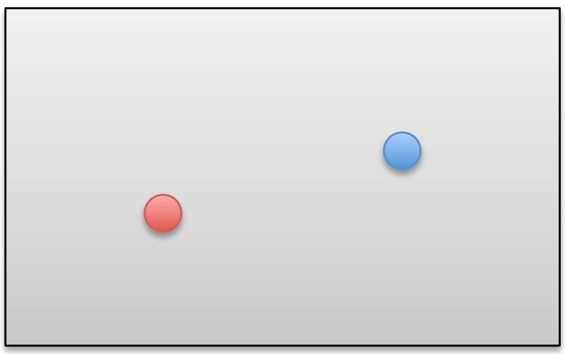
+ time reversal

$\rightarrow T^2 = -1$

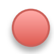
Read, Sachdev, 1991; Wen 1991

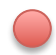
Given topological order + symmetry, what are the possible symmetry fractionalization patterns?

- Z_2 spin liquid



 Bosonic gauge charge

 Bosonic gauge flux

 x  = 

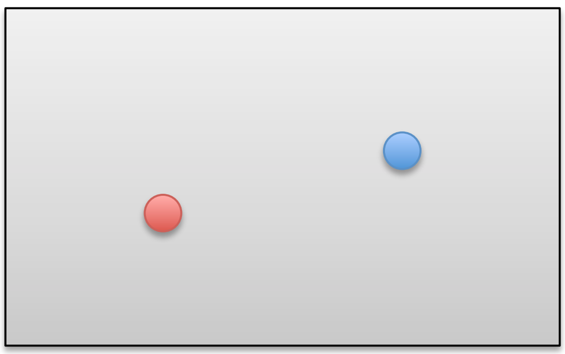
+ time reversal

T^2	A	B	C	D
	1	-1	1	-1
	1	1	-1	-1
	1	-1	-1	1

Wen, 2002; Kitaev, 2006; Essin, Hermele 2013; Mesaros, Ran, 2013; Hung, Wen, 2013; Lu, Vishwanath, 2013; Gu, Hung, Wan, 2014; Barkeshili, Bonderson, Cheng, Wang, 2014; Fidkowski, Lindner, Kiteav.

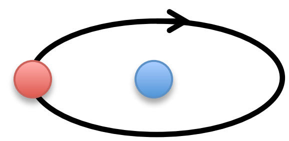
Symmetry fractionalization

- Twisted Z_2 spin liquid



Bosonic gauge charge

Semionic gauge flux $\theta = \frac{\pi}{2}$ $\boxed{s}$

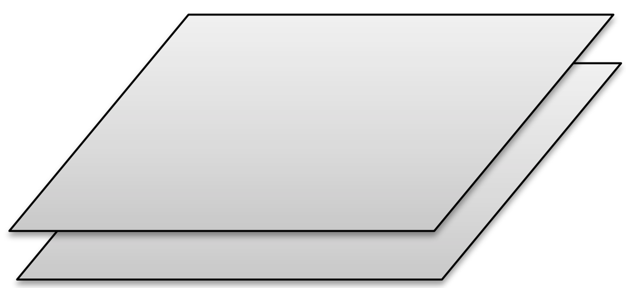


= (-1)

x = $\theta = -\frac{\pi}{2}$ $\boxed{s'}$

$\nu = 1/2$

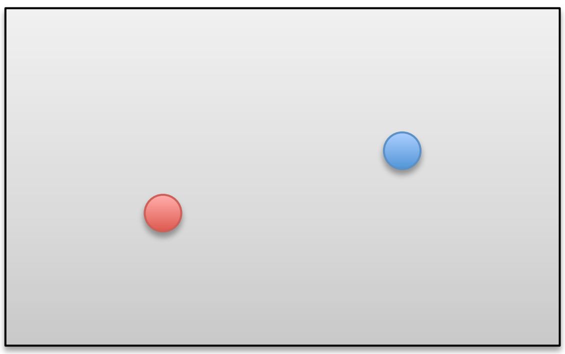
$\nu = -1/2$

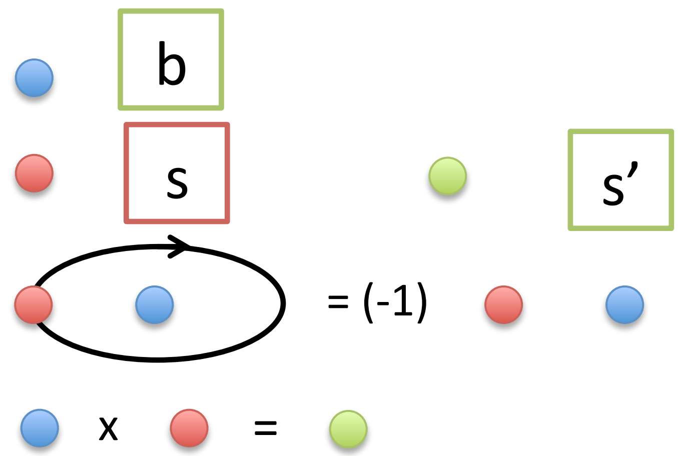


Bosonic
Fractinal
quantum Hall

Symmetry fractionalization

- Twisted Z_2 spin liquid

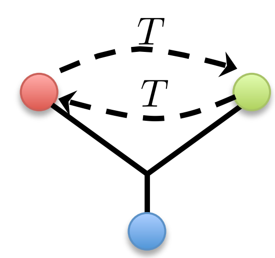




Time Reversal

$$\theta = \frac{\pi}{2} \qquad \theta = -\frac{\pi}{2}$$

$$T^2 = 1$$
$$= e^{i\theta(s,s')}$$

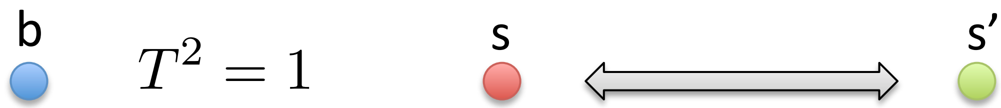


Unique in the bulk

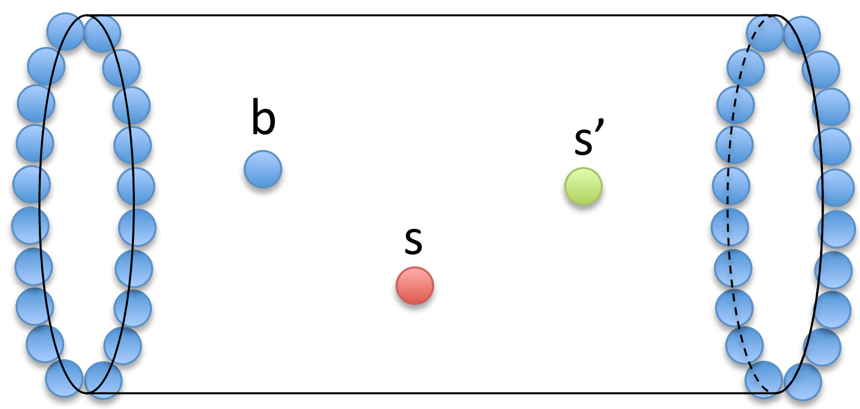
Wang, Potter, Senthil, 2013

Symmetry fractionalization

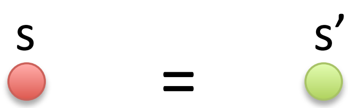
- Twisted Z_2 spin liquid



- More possibilities on the boundary



- Boson condensate
- Gapped
- Time Reversal Symmetric



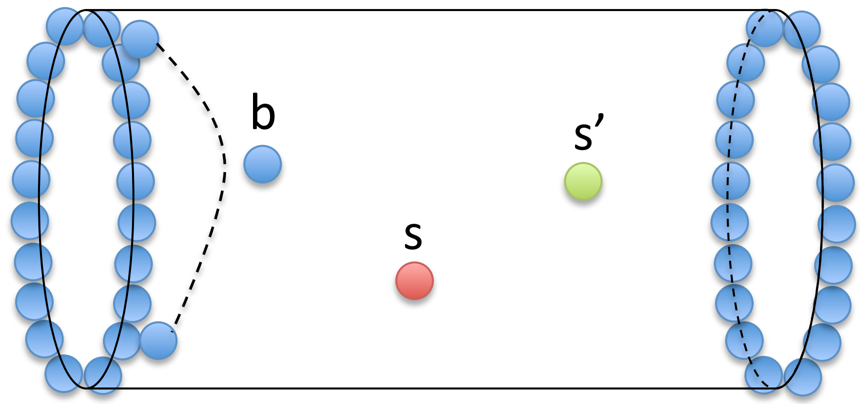
$$T^2 = 1 \text{ or } -1?$$

Summary of result

- Twisted Z_2 gauge theory, gapped boundary with boson condensed
- Two types of time reversal symmetric boundary states
- s(s') transform as $T^2 = 1$ or $T^2 = -1$respectively
- Related to phase of boson condensate
- Simple argument; field theory; exact model
- Domain wall degeneracy

Boson condensate on boundary

- Z_2 boson condensate
- Gapped
- Time reversal symmetric



$$|\psi\rangle = \sum_N (e^{i\alpha})^N \sum_{x_1,\ldots,x_{2N}} |x_1,\ldots,x_{2N}\rangle$$

$$e^{i\alpha} = 1 \qquad e^{i\alpha} = -1 \qquad b_i^\dagger b_j|\psi\rangle = |\psi\rangle$$

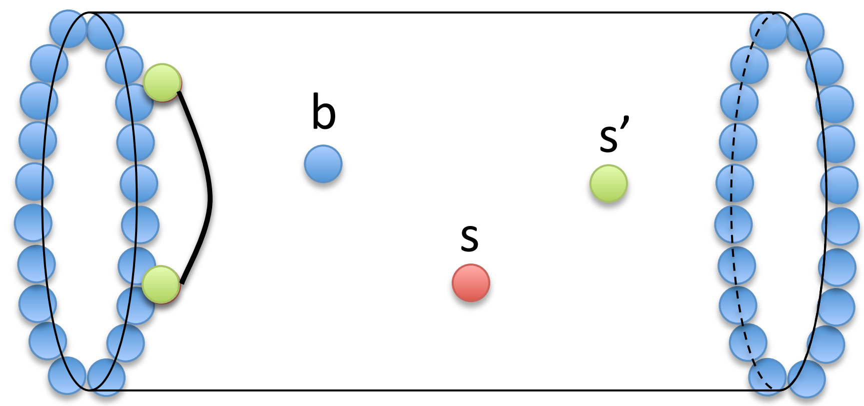
$$\langle b^\dagger\rangle = 1 \qquad \langle b^\dagger\rangle = i \qquad b_i^\dagger b_j^\dagger|\psi\rangle = e^{i\alpha}|\psi\rangle$$

Boson condensate on boundary

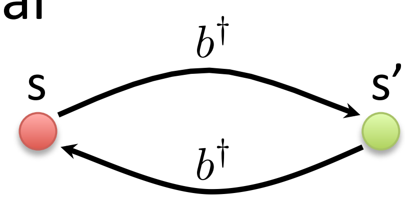
$$|\psi\rangle = \sum_N (e^{i\alpha})^N \sum_{x_1,\ldots,x_{2N}} |x_1,\ldots,x_{2N}\rangle$$

$$e^{i\alpha} = 1 \qquad e^{i\alpha} = -1$$

$$\langle b^\dagger \rangle = 1 \qquad \langle b^\dagger \rangle = i$$



Time Reversal



$$T^2 = \langle b^\dagger b^\dagger \rangle = e^{i\alpha}$$

Time Reversal singlet / doublet

Field Theory Derivation

Twisted Z_2 gauge theory: bulk

$$L = \frac{2}{4\pi}\epsilon^{\lambda\mu\nu} a^1_\lambda \partial_\mu a^1_\nu - \frac{2}{4\pi}\epsilon^{\lambda\mu\nu} a^2_\lambda \partial_\mu a^2_\nu$$

s s'

Boundary

$$L_e = \frac{2}{4\pi}\partial_x \phi_1 \partial_t \phi_1 - \frac{2}{4\pi}\partial_x \phi_2 \partial_t \phi_2$$

Gapped boundary

$$\Delta L = -\lambda \cos(2\phi_1 - 2\phi_2 + \alpha) \qquad \lambda >> 1$$

Condensation of pairs of bosons

Field theory derivation

Boundary

$$L_e = \frac{2}{4\pi}\partial_x\phi_1\partial_t\phi_1 - \frac{2}{4\pi}\partial_x\phi_2\partial_t\phi_2$$

Time reversal symmetry $\qquad \phi_1 \to \phi_2,\ \ \phi_2 \to \phi_1$

Semion on the boundary $\qquad e^{i\phi_1} \qquad e^{i\phi_2}$

$$T^2 \qquad e^{i\phi_1} \to e^{i\phi_2} \to e^{i\phi_1}$$

$$e^{i\phi_2} \to e^{i\phi_1} \to e^{i\phi_2}$$

$$T^2 = 1?$$

Field theory derivation

Boundary

$$L_e = \frac{2}{4\pi}\partial_x\phi_1\partial_t\phi_1 - \frac{2}{4\pi}\partial_x\phi_2\partial_t\phi_2$$

Time reversal symmetry $\quad \phi_1 \to \phi_2, \phi_2 \to \phi_1 + \pi$

Semion on the boundary $\quad e^{i\phi_1} \quad e^{i\phi_2}$

T^2

$$e^{i\phi_1} \to e^{i\phi_2} \to -e^{i\phi_1}$$

$$e^{i\phi_2} \to -e^{i\phi_1} \to -e^{i\phi_2}$$

$$T^2 = -1?$$

Field theory derivation

Boundary

$$L_e = \frac{2}{4\pi}\partial_x\phi_1\partial_t\phi_1 - \frac{2}{4\pi}\partial_x\phi_2\partial_t\phi_2$$

$$\mathcal{T}_1 : \phi_1 \to \phi_2, \phi_2 \to \phi_1$$

$$\mathcal{T}_2 : \phi_1 \to \phi_2, \phi_2 \to \phi_1 + \pi$$

Differ by gauge transformation

$$g : \phi_1 \to \phi_1 + \pi, \phi_2 \to \phi_2$$

Which one? Depends on details of boundary condition

Field theory derivation

Gapped boundary

$$\Delta L = -\lambda \cos(2\phi_1 - 2\phi_2 + \alpha) \qquad \lambda >> 1$$

$\alpha = 0 \qquad \phi_1 - \phi_2 = 0,\ \pi$

Invariant under $\mathcal{T}_1 : \phi_1 \to \phi_2, \phi_2 \to \phi_1$

$$e^{i\phi_1}, e^{i\phi_2}\ T^2 = 1$$

$\alpha = \pi \qquad \phi_1 - \phi_2 = \dfrac{\pi}{2},\ -\dfrac{\pi}{2}$

Invariant under $\mathcal{T}_2 : \phi_1 \to \phi_2, \phi_2 \to \phi_1 + \pi$

$$e^{i\phi_1}, e^{i\phi_2}\ T^2 = -1$$

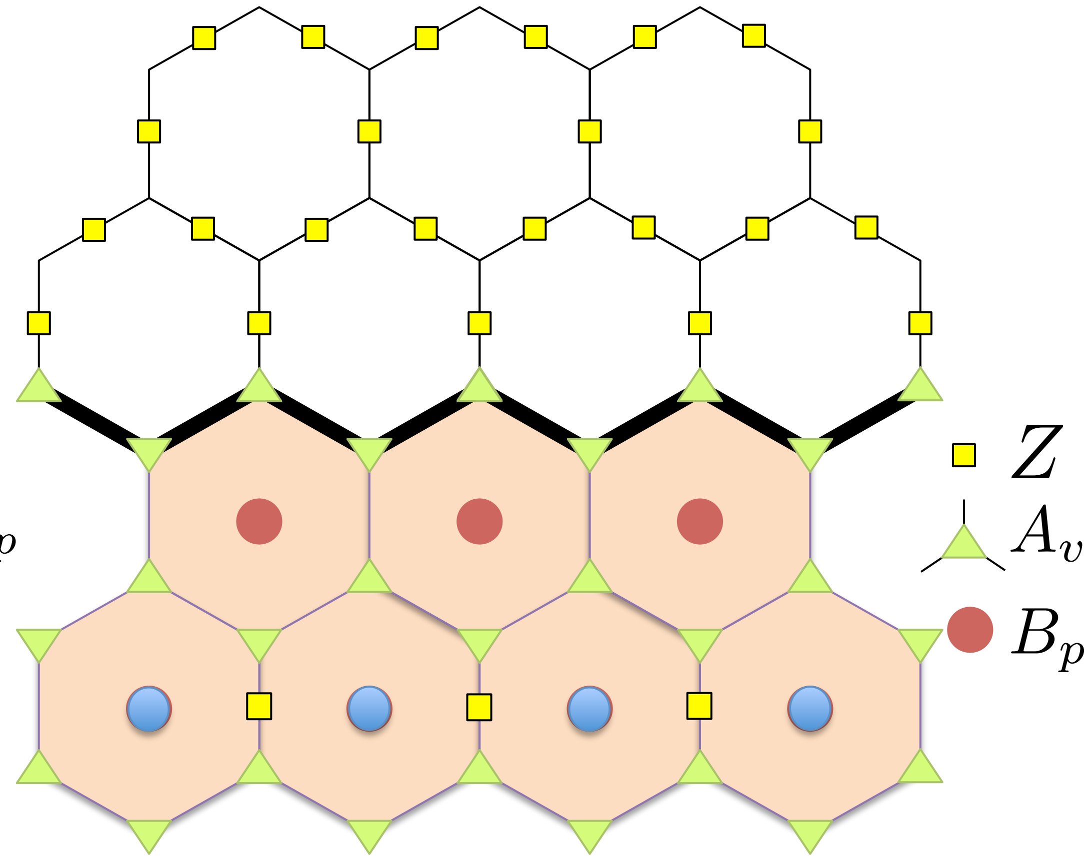
Exactly solvable models

$$H = -\sum_l Z_l$$

Boson condensate

$$H = -\sum_v A_v - \sum_p B_p$$

Gauge theory

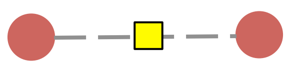


$$|\psi\rangle = \sum_N \sum_{x_1,\ldots,x_{2N}} |x_1,\ldots,x_{2N}\rangle$$

Levin, Wen, 2005

Exactly solvable models

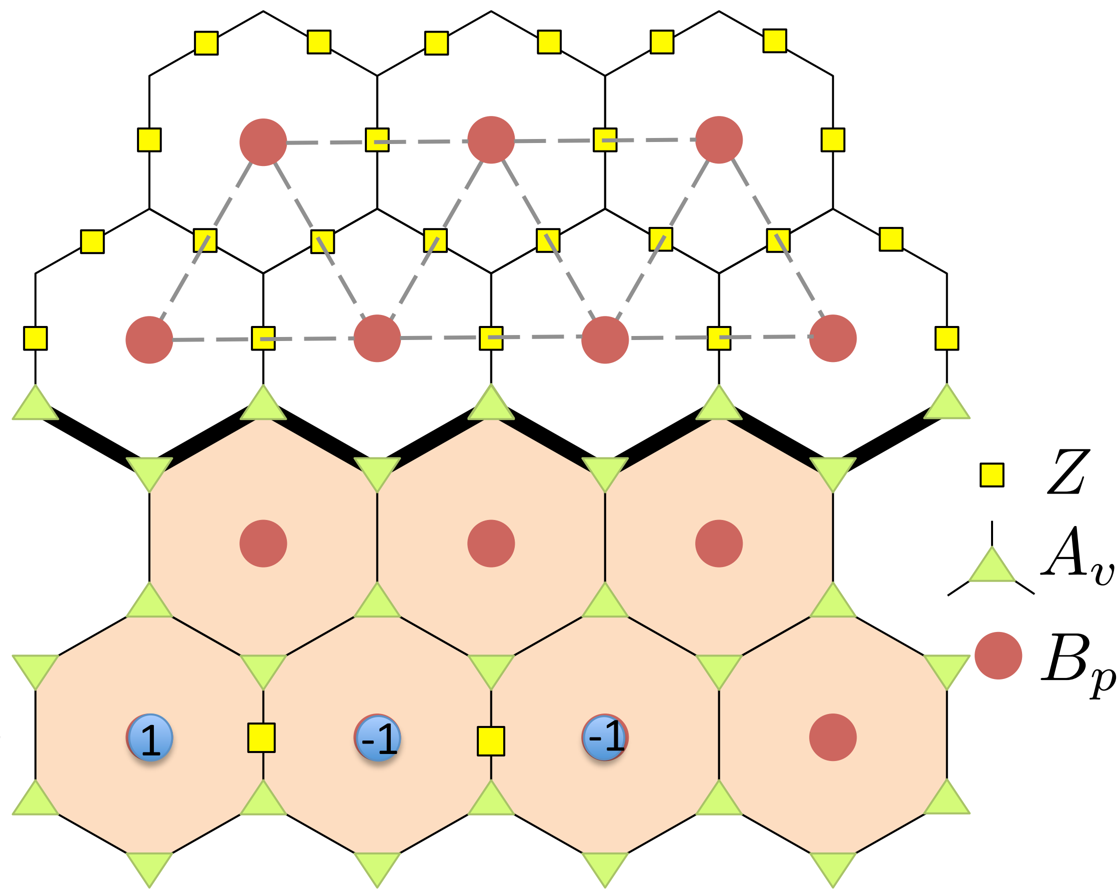
$$H = -\sum_{ab} B_P^a Z_{ab} B_P^b$$



Pair creation -1

Boson hopping 1

$$H = -\sum_v A_v - \sum_p B_p$$

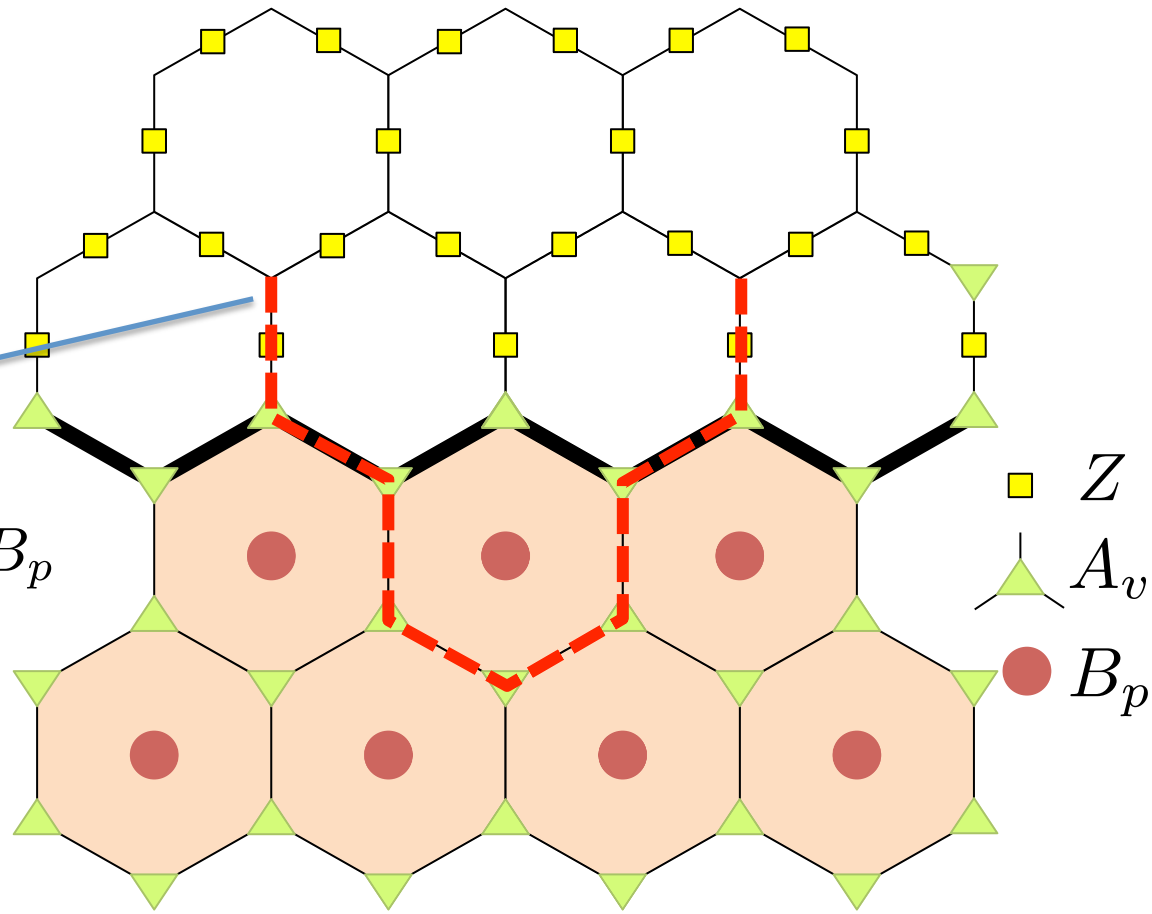


$$|\psi\rangle = \sum_N (-)^N \sum_{x_1,\ldots,x_2 N} |x_1, \ldots, x_2 N\rangle$$

Levin, Wen, 2005

Exactly solvable models

$$H = -\sum_l Z_l$$



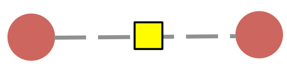
$$H = -\sum_v A_v - \sum_p B_p$$

$$|\psi\rangle = \sum_N \sum_{x_1,\ldots,x_{2N}} |x_1, \ldots, x_{2N}\rangle$$

Levin, Wen, 2005

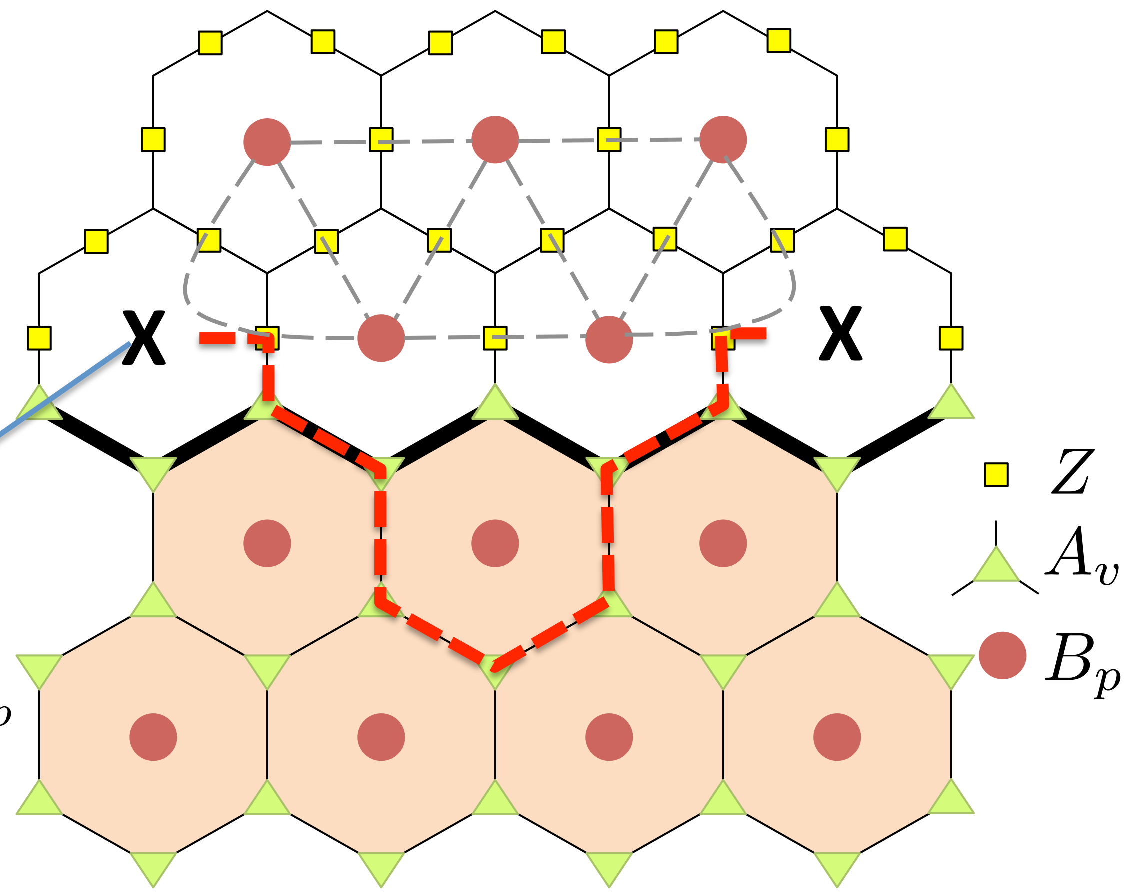
Exactly solvable models

$$H = -\sum_{ab} B_P^a Z_{ab} B_P^b$$



time reversal doublets

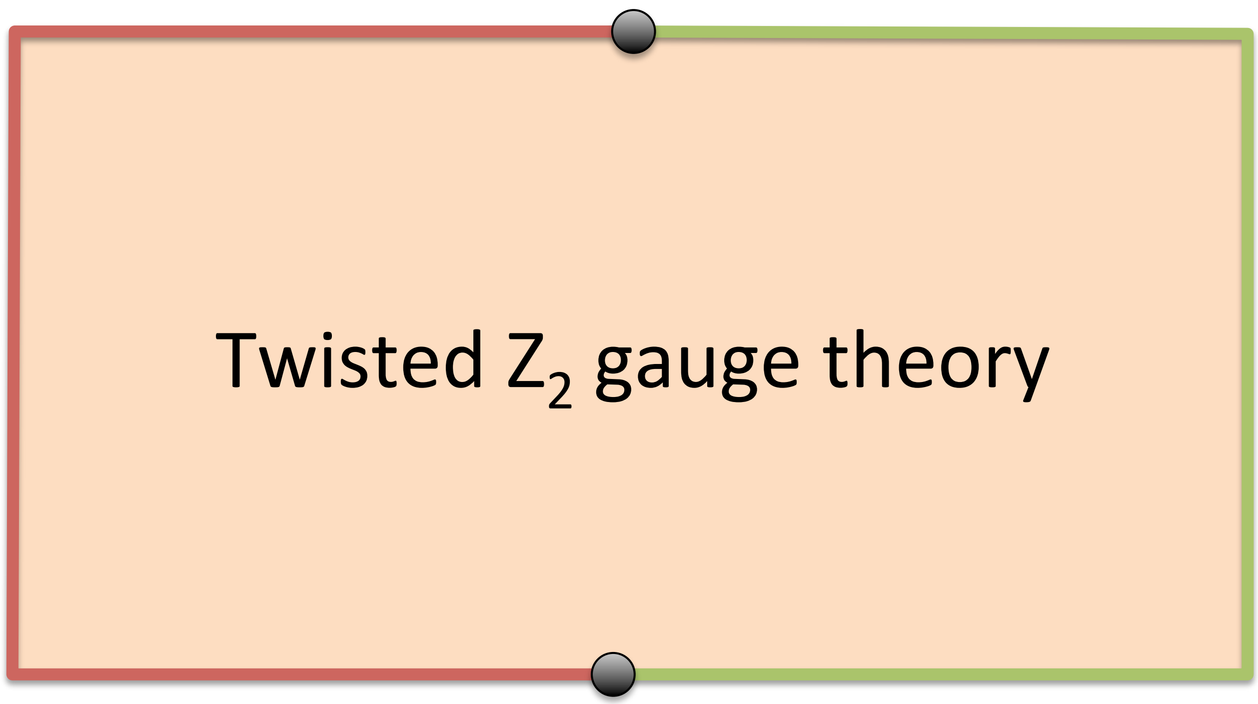
$$H = -\sum_v A_v - \sum_p B_p$$



$$|\psi\rangle = \sum_N (-)^N \sum_{x_1,\ldots,x_2 N} |x_1, \ldots, x_2 N\rangle$$

Levin, Wen, 2005

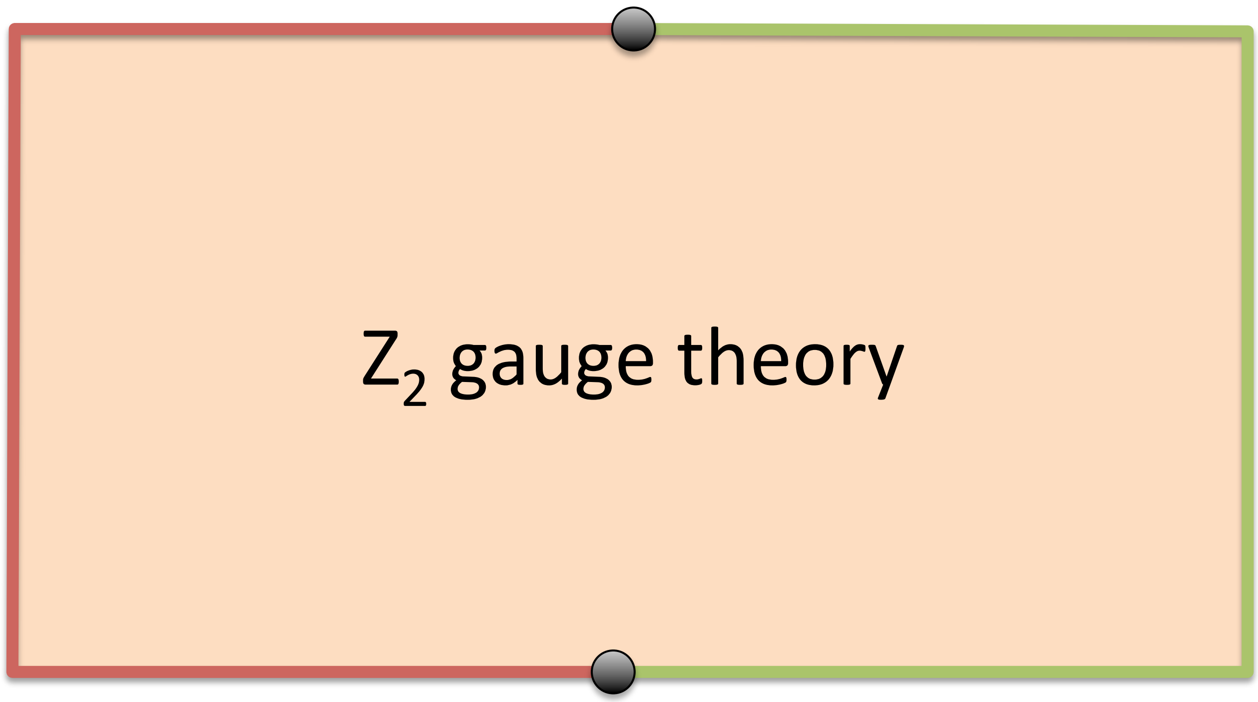
Domain wall degeneracy



$$e^{i\alpha} = 1 \qquad e^{i\alpha} = -1$$

2 fold degeneracy associated with each domain wall

Domain wall degeneracy



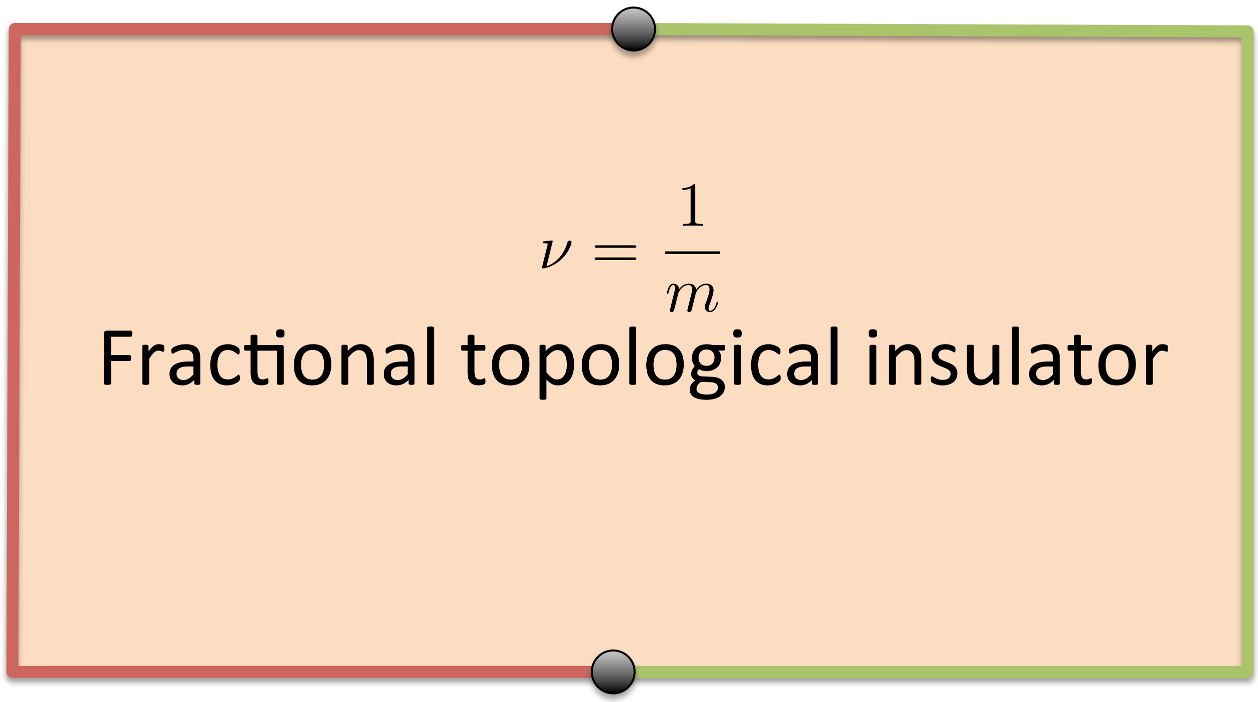
gauge flux condensing   gauge charge condensing

$\sqrt{2}$ fold degeneracy associated with each domain wall

No symmetry required

Bravyi, Kitaev, 1998

Domain wall degeneracy



$\sqrt{2m}$ fold degeneracy associated with each domain wall

Fu, Kane, 2009; Cheng 2012; Clarke, Alicea, Shtengel, 2013; Lindner, Berg, Refael, Stern, 2012; Vaezi, 2013

Domain wall degeneracy

$$e^{i\alpha} = 1$$

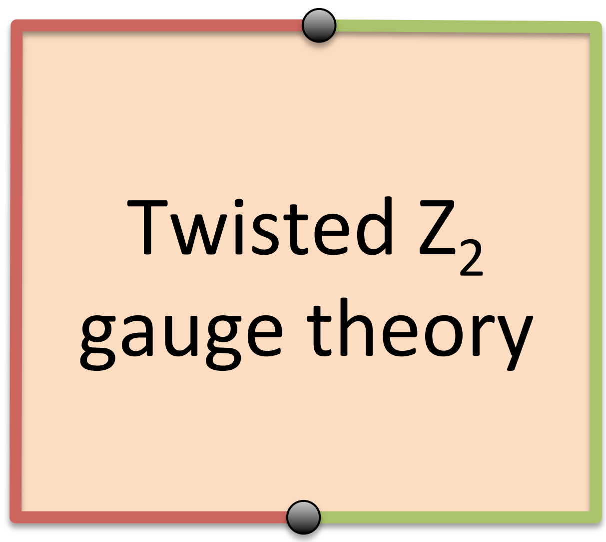
$$-\cos(2\phi_1 - 2\phi_2)$$

$$\phi_1 - \phi_2 = 0$$

$$|A\rangle$$

$$\phi_1 - \phi_2 = \pi$$

$$|\bar{A}\rangle$$



$$e^{i\alpha} = -1$$

$$\cos(2\phi_1 - 2\phi_2)$$

$$\phi_1 - \phi_2 = \pi/2$$

$$|B\rangle$$

$$\phi_1 - \phi_2 = -\pi/2$$

$$|\bar{B}\rangle$$

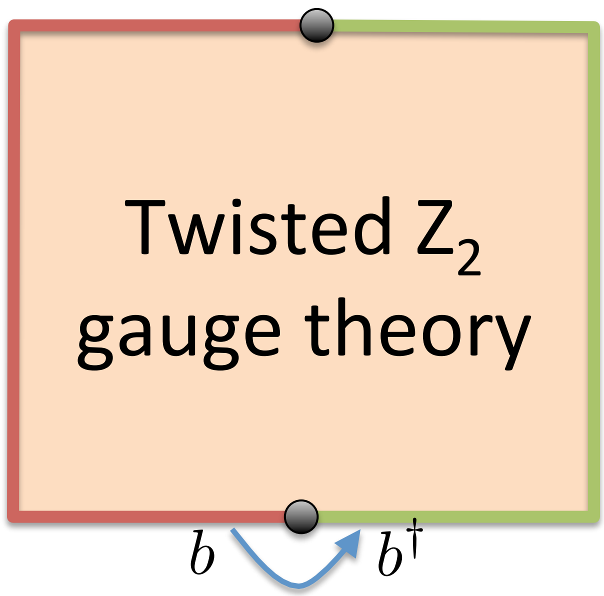
Z_2 gauge transformation $g : \phi_1 \to \phi_1 + \pi, \phi_2 \to \phi_2$

$$|A\rangle \leftrightarrow |\bar{A}\rangle \qquad |B\rangle \leftrightarrow |\bar{B}\rangle$$

Degenerate edge space $|AB\rangle + |\bar{A}\bar{B}\rangle \quad |A\bar{B}\rangle + |\bar{A}B\rangle$

Domain wall degeneracy

$$\phi_1 - \phi_2 = 0$$
$$|A\rangle$$
$$\phi_1 - \phi_2 = \pi$$
$$|\bar{A}\rangle$$



$$\phi_1 - \phi_2 = \pi/2$$
$$|B\rangle$$
$$\phi_1 - \phi_2 = -\pi/2$$
$$|\bar{B}\rangle$$

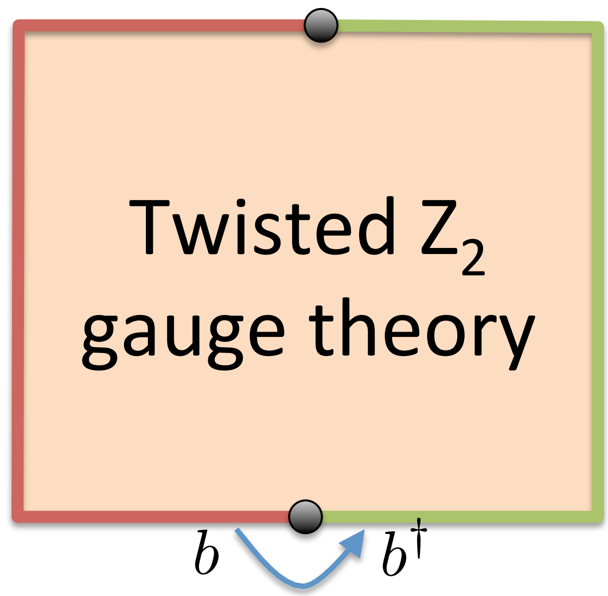
Degenerate edge space $|AB\rangle + |\bar{A}\bar{B}\rangle \quad |A\bar{B}\rangle + |\bar{A}B\rangle$

$$(-ib_A b_B^\dagger + h.c.)(|AB\rangle + |\bar{A}\bar{B}\rangle) = (|AB\rangle + |\bar{A}\bar{B}\rangle)$$
$$(-ib_A b_B^\dagger + h.c.)(|A\bar{B}\rangle + |\bar{A}B\rangle) = -(|A\bar{B}\rangle + |\bar{A}B\rangle)$$

Breaks time reversal

Domain wall degeneracy

$$\phi_1 - \phi_2 = 0$$
$$|A\rangle$$
$$\phi_1 - \phi_2 = \pi$$
$$|\bar{A}\rangle$$



$$\phi_1 - \phi_2 = \pi/2$$
$$|B\rangle$$
$$\phi_1 - \phi_2 = -\pi/2$$
$$|\bar{B}\rangle$$

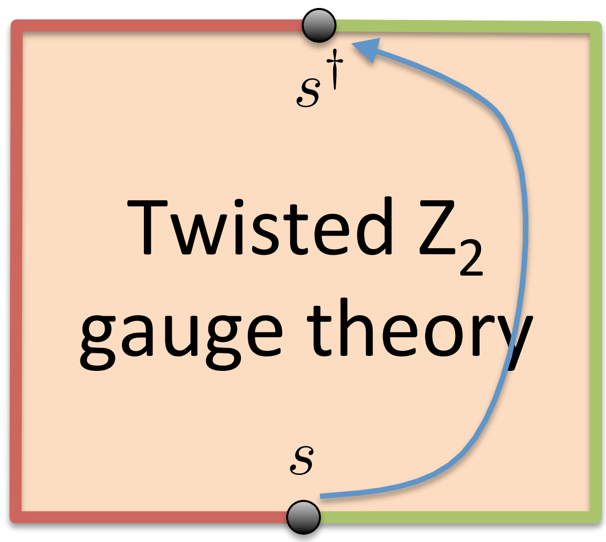
Degenerate edge space $|AB\rangle + |\bar{A}\bar{B}\rangle \quad |A\bar{B}\rangle + |\bar{A}B\rangle$

$$\mathcal{T}_1 : \phi_1 \to \phi_2, \phi_2 \to \phi_1$$
$$\mathcal{T}_2 : \phi_1 \to \phi_2, \phi_2 \to \phi_1 + \pi$$

$$(-ib_A b_B^\dagger + h.c.) \to -(-ib_A b_B^\dagger + h.c.)$$

Domain wall degeneracy

$$\phi_1 - \phi_2 = 0$$
$$|A\rangle$$
$$\phi_1 - \phi_2 = \pi$$
$$|\bar{A}\rangle$$



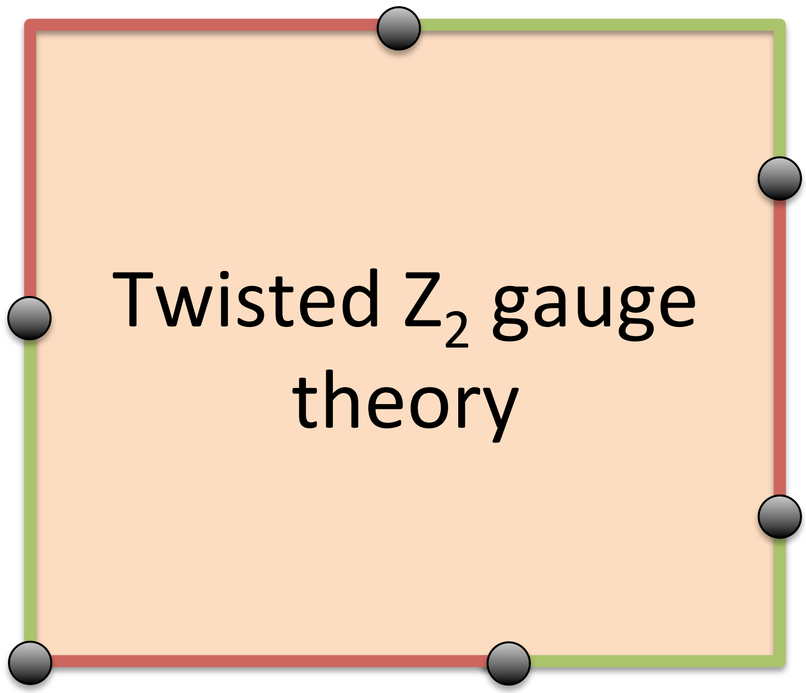
$$\phi_1 - \phi_2 = \pi/2$$
$$|B\rangle$$
$$\phi_1 - \phi_2 = -\pi/2$$
$$|\bar{B}\rangle$$

Degenerate edge space $|AB\rangle + |\bar{A}\bar{B}\rangle \quad |A\bar{B}\rangle + |\bar{A}B\rangle$

$$W_s : |AB\rangle + |\bar{A}\bar{B}\rangle \leftrightarrow |A\bar{B}\rangle + |\bar{A}B\rangle$$ Nonlocal

$$T : |AB\rangle + |\bar{A}\bar{B}\rangle \leftrightarrow |A\bar{B}\rangle + |\bar{A}B\rangle$$

Domain wall degeneracy



Time reversal symmetry protected degeneracy 2^{N-1}

Summary

- Twisted Z_2 gauge theory, gapped boundary with boson condensed
- Two types of time reversal symmetric boundary states
- s(s') transform as $T^2=1$ or $T^2=-1$respectively
- Depending on the ± 1 pair creation phase of the boson condensate
- Two fold degeneracy on domain wall

Relation to Symmetry Protected Topological Phases

$Z_2 \times Z_2^T$ SPT Phases	T symmetric Twisted Z_2 gauge theory
• Z_2 symmetry breaking gapped edge	• Boson condensed gapped edge
• $T^2 = \pm 1$ on Z_2 domain wall	• $T^2 = \pm 1$ on semion
• two bulk phases	• One bulk phase
Phase A $\Longleftrightarrow$ Phase B	$\{I, g, T, gT\}$

Gauge Z_2: $Z_2 \times Z_2^T$ SPT Phases $\Longrightarrow$ T symmetric Twisted Z_2 gauge theory

Conclusion

- Different symmetric edge for the same symmetric bulk
- Applies more generally
- Realization